Forme algébrique d'un nombre complexe

Exercice 1

Déterminer la forme algébrique des nombres complexes ci-dessous :

1. $2i + 3 - 5i - 1 = 2i - 5i + 3 - 1 = -3i + 2 = 2 - 3i$
2. $4 - 3i + 7 + i - 7i = 4 + 7 - 3i + i - 7i = 11 - 9i$
3. $12 + 8i - 8 - 6i = 12 - 8 + 8i - 6i = 4 + 2i$
4. $3 + i - 2 + i = 3 - 2 + i + i = 1 + 2i$
5. $(2 + 3i)(1 + i) = 2 + 2i + 3i + 3i^2 = 2 + 5i - 3 = -1 + 5i$
6. $3 - 5i - (2 + i) = 3 - 5i - 2 - i = 3 - 2 - 5i - i = 1 - 6i$
7. $(4 - 3i)(2 - 3i) = 8 - 12i - 6i + 9i^2 = 8 - 18i - 9 = -1 - 18i$
8. $3(2 - i) - 2(1 + 2i) = 6 - 3i - 2 - 4i = 6 - 2 - 3i - 4i = 4 - 7i$
9. $(1 + 4i)(1 - i) = 1 - i + 4i - 4i^2 = 1 + 3i + 4 = 5 + 3i$
10. $2(4 + i) - 3(2i + 1) = 8 + 2i - 6i - 3 = 8 - 3 + 2i - 6i = 5 - 4i$

Exercice 2

Déterminer la forme algébrique des nombres complexes ci-dessous :

1. $(1 + i)^2 = 1^2 + 2(1)(i) + i^2 = 1 + 2i - 1 = 2i$
2. $(1 - i)^2 = 1^2 - 2(1)(i) + i^2 = 1 - 2i - 1 = -2i$
3. $(1 + i)^3 = 1^3 + 3(1)^2(i) + 3(1)(i)^2 + i^3 = 1 + 3i - 3 - i = -2 + 2i$
4. $(1 - i)^3 = 1^3 - 3(1)^2(i) + 3(1)(i)^2 - i^3 = 1 - 3i - 3 + i = -2 - 2i$
5. $(1 + i)^4 = 1^4 + 4(1)^3(i) + 6(1)^2(i)^2 + 4(1)(i)^3 + i^4 = 1 + 4i - 6 - 4i + 1$

   $(1 + i)^4 = 1 - 6 + 1 + 4i - 4i = -4$

   Autre méthode

   $(1 + i)^2 = 2i$, donc : $(1 + i)^4 = [\,(1 + i)^2]^2 = (2i)^2 = 4(i)^2 = -4$
6. $(1 - i)^4 = [\,(1 - i)^2]^2 = (-2i)^2 = 4(i)^2 = -4$
7. $(1 + i)^5 = (1 + i)^2(1 + i)^3 = 2i(-2 + 2i) = -4i + 4i^2 = -4 - 4i$

   Autre méthode

   $(1 + i)^5 = 1^5 + 5(1)^4(i) + 10(1)^3(i)^2 + 10(1)^2(i)^3 + 5(1)(i)^4 + i^5$

   $(1 + i)^5 = 1 + 5i - 10 - 10i + 5 + i = 1 - 10 + 5 + 5i - 10i + i = -4 - 4i$
8. $(1 - i)^5 = (1 - i)^2(1 - i)^3 = -2i(-2 - 2i) = 4i + 4i^2 = -4 + 4i$
9. $1 + i + i^2 + i^3 = 1 + i - 1 - i = 0$

10. $1 + i + i^2 + i^3 + i^4 = i^4$

11. $\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^2 = \left(\frac{\sqrt{2}}{2}(1+i)\right)^2 = \left(\frac{\sqrt{2}}{2}\right)^2 (1+i)^2 = \frac{1}{2}(2i) = i$

12. $\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^4 = \left[\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^2\right]^2 = i^2 = -1$

13. $\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^2 = \left(\frac{1}{2}(1+\sqrt{3}i)\right)^2 = \left(\frac{1}{2}\right)^2 (1+\sqrt{3}i)^2 = \frac{1}{4}\left(1^2 + 2(1)(\sqrt{3}i) + \left(\sqrt{3}i\right)^2\right)$

$\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^2 = \frac{1}{4}\left(1 + 2\sqrt{3}i - 3\right) = \frac{1}{4}\left(-2 + 2\sqrt{3}i\right) = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

14. $\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^3 = \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^2 \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\left(\left(\frac{1}{2}\right) + \frac{\sqrt{3}}{2}i\right) = \left(\frac{\sqrt{3}}{2}i\right)^2 - \left(\frac{1}{2}\right)^4$

$\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^3 = -\frac{3}{4} - \frac{1}{4} = -1$

15. $\frac{1}{i} = \frac{i}{i^2} = -i$

16. $\frac{1}{1+i} = \frac{1\times(1-i)}{(1+i)(1-i)} = \frac{1-i}{(1^2-i^2)} = \frac{1-i}{(1+1)} = \frac{1-i}{2} = \frac{1}{2} - \frac{1}{2}i$

17. $\frac{1}{1-i} = \frac{1\times(1+i)}{(1-i)(1+i)} = \frac{1+i}{(1^2-i^2)} = \frac{1+i}{(1+1)} = \frac{1+i}{2} = \frac{1}{2} + \frac{1}{2}i$

18. $\frac{1}{(1+i)^2} = \left(\frac{1}{1+i}\right)^2 = \left(\frac{1}{2} - \frac{1}{2}i\right)^2 = \left(\left(\frac{1}{2}\right)(1-i)\right)^2 = \left(\frac{1}{2}\right)^2 (1-i)^2 = \frac{1}{4}(-2i)$

$\frac{1}{(1+i)^2} = -\frac{1}{2}i$

19. $\frac{1}{1-i} + \frac{1}{1+i} = \frac{(1+i)}{(1-i)(1+i)} + \frac{(1-i)}{(1+i)(1-i)} = \frac{1+i+1-i}{2} = \frac{2}{2} = 1$

20. $\frac{1}{1+i} - \frac{1}{1-i} = \frac{(1-i)}{(1-i)(1+i)} - \frac{(1+i)}{(1+i)(1-i)} = \frac{1-i-1-i}{2} = \frac{-2i}{2} = -i$