Les nombres complexes exercices

Exercice 1

Donner la partie réelle et la partie imaginaire des nombres complexes.

z	Re(z)	Im(z)
-2 + i	-2	1
$\sqrt{5}$	$\sqrt{5}$	0
-2i	0	-2
$4\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right) = -2+2i\sqrt{3}$	-2	$2\sqrt{3}$
$\frac{1-4i}{2} = \frac{1}{2} - 2i$	$\frac{1}{2}$	-2

Exercice 2

Écrire les nombres suivants sous forme algébrique :

$a = 2(1-3i) - (1-i) = 2 - 6i - 1 + i = 1 - 5i$

$b = -4i^2 + i = 4 + i$

$c = i(-2+7i) = -2i - 7 = -7 - 2i$

$d = \frac{1}{i} = \frac{i}{i^2} = -i$

$e = i^3 = i^2 \times i = -i$

$f = i^4 = i^2 \times i^2 = -1 \times (-1) = 1$

$g = i^5 = i^2 \times i^3 = -1 \times (-i) = i$

$h = i^6 = (i^2)^3 = (-1)^3 = -1$

Exercice 3

Écrire les nombres suivants sous forme algébrique :

$a = (1+i)(1-i) = 1^2 - i^2 = 1 + 1 = 2$

$b = (2+i)^2 = 2^2 + 2(2)(i) + i^2 = 4 - 1 + 4i = 3 + 4i$

$c = \left(\sqrt{2} - i\sqrt{3}\right)\left(\sqrt{2} + i\sqrt{3}\right) = \sqrt{2}^2 - \left(i\sqrt{3}\right)^2 = 2 + 3 = 5$

$d = (2 + 3i)^2 = 2^2 + 2(2)(3i) + (3i)^2 = 4 - 9 + 12i = -5 + 12i$

Exercice 4

On pose $z_1 = -6 + 12i$ et $z_2 = 4 - 12i$.

Donner la forme algébrique des nombres suivants :

$-\frac{1}{3}z_1 = -\frac{1}{3}(-6 + 12i\ ) = 2 - 4i$

$\frac{1}{4}z_2 = \frac{1}{4}(4 - 12i) = 1 - 3i$

$2z_1 + 3z_2 = 2(-6 + 12i) + 3(4 - 12i) = -12 + 24i + 12 - 36i = -12i$

$\frac{1}{12}z_1z_2 = \frac{1}{12}(-6 + 12i\ )(4 - 12i) = (-2 + 4i\ )(1 - 3i) = -2 + 12 + 4i + 6i$

$\frac{1}{12}z_1z_2 = 10 + 10i$

$z_1 + z_2 = -6 + 12i + 4 - 12i = -2$

$\frac{z_1}{z_2} = \frac{-6 + 12i}{4 - 12i} = \frac{-3 + 6i}{2 - 6i} = \frac{3}{2} \times \frac{-1 + 2i}{1 - 3i} = \frac{3}{2} \times \frac{(-1 + 2i\ )(1 + 3i)}{10}$

$\frac{z_1}{z_2} = \frac{3}{2} \times \frac{-1 - 6 + 2i - 3i}{10} = \frac{3}{2} \times \frac{-7 - i}{10} = \frac{-21}{20} - \frac{3}{10}i$

Exercice 5

On pose $z = 2 + i$ et $z' = 3 - 2i$.

Donner la forme algébrique des nombres suivants :

$z + z' = 2 + i + 3 - 2i = 5 - i$

$zz' = (2 + i)(3 - 2i) = 6 + 2 + 3i - 4i = 8 - i$

$z'^2 = (3 - 2i)^2 = 3^2 - 2(3)(2i) + (2i)^2 = 9 - 4 - 12i = 5 - 12i$

$(z + z')^2 = (5 - i)^2 = 5^2 - 2(5)(i) + (i)^2 = 25 - 1 - 10i = 24 - 10i$

$\frac{z}{z'} = \frac{(2 + i)}{(3 - 2i)} = \frac{(2 + i)(3 + 2i)}{13} = \frac{6 - 2 + 3i + 4i}{13} = \frac{4 + 7i}{13} = \frac{4}{13} + \frac{7}{13}i$

$\frac{z'}{z} = \frac{13}{4 + 7i} = \frac{13(4 - 7i)}{16 + 49} = \frac{13(4 - 7i)}{65} = \frac{(4 - 7i)}{5} = \frac{4}{5} - \frac{7}{5}i$