In English - exercises 79 et 80

79 Sequences by induction

1. Sequence $u_1, u_2, u_3, \ldots$ is defined by:

$$u_1 = 2 \text{ and } u_{n+1} = \frac{5u_n - 3}{3u_n - 1}.$$

Prove by induction that, for all integers $n \geqslant 1$,

$$u_n = \frac{3n+1}{3n-1}.$$

2. Sequence $u_1, u_2, u_3, \ldots$ is defined by:

$$u_1 = \frac{3}{4} \text{ and } u_{n+1} = \frac{3}{4 - u_n}.$$

Prove by induction that, for all integers $n \geqslant 1$,

$$u_n = \frac{3^{n+1} - 3}{3^{n+1} - 1}.$$

80 Sequences by algebraic form

1. Express $(k+1)^2 + 5(k+1) + 8$ in the form $k^2 + ak + b$, where a and b are constants.

2. Prove by induction that, for all integers $n \geqslant 1$,

$$\sum_{r=1}^{n} r(r+1)\left(\frac{1}{2}\right)^{r-1} = 16 - (n^2 + 5n + 8)\left(\frac{1}{2}\right)^{n-1}.$$